How Children's Understanding of the Number System varies as a Function of Ethnicity and Socio-Economic Status

Jennifer Young-Loveridge
University of Waikato
<educ2233@waikato.ac.nz>

This study looked at Year 3 children's understanding of the number system at four schools with varying socio-economic status. Just over one third of the children were Maori (the indigenous people of New Zealand). In total, 221 children were interviewed using a task-based interview. Children's understanding of the number system varied as a function of ethnicity and socio-economic status. Performance on several key tasks was used to assign children to a level on a developmental framework for numeracy.

A little over a year ago, a new literacy and numeracy goal for young New Zealanders was announced by the Minister of Education. The goal stated that "by 2005, every child turning 9 will be able to read, write and do maths for success" (Ministry of Education, 1999a). It is not clear just what "doing maths for success" meant, or how it was to be measured. However, it is unlikely that this goal could be met without having a good understanding the number system and the ability to apply this knowledge to problems within a variety of contexts (including measurement, statistics, algebra, and geometry). In view of the enormous variation in children's understanding about numbers when they enter school on their fifth birthday (see Young-Loveridge, 1989, 1991), it seems clear that some children will require a great deal of extra assistance if they are to met the numeracy goal by their ninth birthday. In order to identify which children are in particular need of such assistance, appropriate assessment tools will need to be developed.

Assessment has been a particularly controversial issue in New Zealand over the last few years. In May 1998, the Government published a Green Paper: *Assessment for Success in Primary Schools,* proposing the introduction of national externally-referenced tests (Ministry of Education, 1999b). There was a three month period of consultation during which the public could make written submissions to the Ministry of Education (Gilmore, 1999). Responses to the Green paper showed overwhelming opposition to the idea of introducing national externally-referenced tests, with more than 14,000 reasons given for this position. However, there was strong support for the development of additional diagnostic tests for use in primary schools. A call was made for these diagnostic tests be closely related to the curriculum, valid, reliable, culturally appropriate, easy to administer and not too demanding of time. Mathematics was identified as one of the curriculum areas in which the development of diagnostic tests was most urgently required (Gilmore, 1999).

Informal discussions with teachers over the past several years has consistently included concern about the need for a model of developmental progression in mathematics. Several researchers have developed models to explain how children's understanding about numbers develop as they progress from beginning to competent thinking (eg, Bergeron & Herscovics, 1990; Boulton-Lewis, 1996; Fuson, Smith & Cicero, 1997a; Fuson et al, 1997b; Hiebert, 1988; Jones et al, 1996; Resnick, 1983; Ross, 1989; Thomas, 1996; Wright, 1996). However, each of these models has problems or limitations, which include being focused on a very narrow domain of understanding, being overly complicated and difficult to apply quickly in a classroom context, or lacking a clear rationale for progress to more advanced stages. Recently a framework was developed to overcome some of those problems and to provide teachers with a tool which could assist them in identifying strengths and weaknesses in children's understanding about the number system and meet the learning needs of these students more effectively (see Young-Loveridge, 1999a).The framework consists of four stages, each characterised by a major shift in ways of thinking about numbers (see Figure 1). The framework shows how children's understanding of the number system becomes increasingly sophisticated as their thinking develops. The framework begins with a unitary (by ones) concept of numbers, moves through a transitional stage of ten-structured thinking, leading to a multi-unit (by tens and ones) concept of numbers, and finishing with an extended multi-unit stage where multiple units can include any power of ten.

1. **Unitary concept**
   Knowledge of number word sequences, processes for constructing quantities, ways of working with part-whole relationships, and names for numerals and number patterns

2. **Ten-structured concept**
   Partitioning of numbers into a whole decade and extra ones, and partitioning of the whole decade into units of 10 ones

3. **Multi-unit concept**
   Units of tens and ones are counted separately, and can be traded and exchanged for units of different value (eg, 10 ones for one 10, or vice versa).

4. **Extended multi-unit concept**
   Units can be any power of ten.

Figure 1. Developmental framework for the acquisition of numeracy.

This study was designed to explore children's understanding about the number system during their third year at school in order to identify those with limited understanding about the number system who might benefit from intervention. The tasks used to assess children's understanding of the number system were developed also with the aim of providing diagnostic tools which might be useful for teachers in primary schools. As part of the study, children were assigned to one of the first three levels of the number framework (Young-Loveridge, 1999a).

Method

Participants

The children who participated in this study came from Year 3 classes in four schools in a large urban centre of New Zealand. There were 221 children in total, with approximately one third of them Maori (the indigenous people of Aotearoa, New Zealand), half of them European, and the remainder from other ethnic groups (see Table 1). The socio-economic status of one of the four schools was average (i.e., School A was at level 5 of a 10-point scale ranging from 1 for low decile to 10 for high decile), while the SES of the other three schools was low (Schools B & C were at decile 1 and School D was at decile 2). The group of children from School A were the youngest in their year group (average age = 7.6 years), whereas the groups of children from Schools B, C, and D included all of the children in Year 3 attending those schools (average age = 7.9, 8.0 and 8.0 years, respectively). Because of School C's small size and its similarity to School B, the two decile 1 schools were subsequently combined for the purposes of statistical analysis.

Table 1
Percentages of Children in each Group

	School			
	A	B	C	D
School's Decile Level	5	1	1	2
Group Size	73	52	19	77
Ethnicity				
European/Pakeha	62	10	26	56
NZ Maori	22	65	63	33
Other	16	25	11	11
Gender				
Boys	41	50	58	49
Girls	59	50	32	51
Age in years				
Range	7.1 to 8.1	7.3 to 8.3	7.5 to 8.5	7.4 to 8.6
Average	7.6	7.9	8.0	8.0
SD	0.28	0.29	0.34	0.28
Level				
Unitary	30	62	47	38
Ten-Structured	34	30	53	43
Multi-unit	36	8	0	19

Procedure

An individual task-based interview was used to assess the number knowledge of each child. Tasks included reading and writing numerals, constructing quantities using grouped materials and money of different denominations ($1, $10, $100), part-whole understanding including mental operations, written computation, and word problems, demonstration of place-value understanding, and knowledge of number-word sequences. The strategies children used to solve the problems were noted, in addition to the accuracy of their responses. Children's performance on several key tasks was used to assign them to a level within a framework showing developmental progression in children's understanding of the number system (see Young-Loveridge, 1999a). The task which was used to differentiate children at the Unitary level from those at the higher levels was one in which children were asked to imagine that they had 20 lollies and were given 8 more lollies. Children assigned to the Ten-structured level or higher could say immediately that there were 28 lollies in total, whereas those at the Unitary level had to count by ones to determine the sum of 20 and 8. A collection of tasks used to differentiate children at the Ten-structured level from those at the Multi-unit level involved demonstrating the link between individual digits in a multi-digit numeral and the quantities these digits represented. Sharon Ross's (1989) task with buttons was modified to create a pencil-and-paper version in which the objects (small boxes printed on paper) were presented in a standard arrangement with boxes organised in groups of ten. Children were asked first to determine how many boxes were presented in a particular display, then to write that number on the line above the display, and finally to circle with a pencil the collection of boxes to which each digit in the multi-digit numeral referred. An example of this three-step process was given to the children using the number "12". Children who responded correctly for each digit in 14, 21, 31 and 125 were assigned to the Multi-unit level of the number framework.

Results and Discussion

The majority of children were at the Unitary Level (42%), or the Ten-structured Level (38%), with only 20% at the Multi-unit Level. Table 2 shows the percentages of children at each level of the framework as a function of ethnicity and socio-economic status.

Table 2
Percentages of Year 3 Children at each Level of the Framework as a Function of Ethnicity and Socio-economic Status

		Ethnicity			
	(Overall)	European	Maori	Other	
School A (decile 5)	(73)	(45)	(9)	(19)	
Unitary Level	(22)	22	78	26	
Ten-Structured Level	(25)	34	15	67	
Multi-unit Level	(26)	44	7	7	
Schools B & C (decile 1)	(71)	(10)	(46)	(15)	
Unitary Level	(41)	20	63	67	
Ten-Structured Level	(26)	60	35	15	
Multi-unit Level	(4)	20	2	7	
School D (decile 2)	(77)	(43)	(25)	(9)	
Unitary Level	(29)	40	28	56	
Ten-Structured Level	(33)	41	52	22	
Multi-unit Level	(15)	19	20	22	
Total		(221)	(98)	(80)	(43)

Note. Numbers are shown in brackets.

In general, children at the average decile school had better understanding than those at the three low decile schools, and Non-Maori children had better understanding than Maori children. However, several notable exceptions to this general pattern were found for School D, a decile 2 school. At School D, there were substantially more children at the Multi-unit level, compared with Schools B and C (15% compared to 4%). At School D, the proportion of Maori children at the Multi-unit level was very similar to that of the Non-Maori children

(20% compared to 19% & 22%), whereas for the other cohorts, the proportion of Maori children was substantially less than for Non-Maori children at that level. At School D (decile 2), only 28% of the Maori children were at the Unitary Level, compared to 63% of Maori children at Schools B and C (decile 1), and 78% at School A, (decile 5).

A comparison of Maori children across the four different schools showed that Maori children at School D did substantially better than those at Schools A, B, or C, with 72% at the Ten-Structured Level or better (compared to 37% and 22% at the decile 1 and 5 schools, respectively). There are many possible reasons for the success of Maori children at School D. School D seems to have a particularly strong commitment to bi-culturalism, with 5 of its 21 classes offering education in Te Reo Maori (the Maori language). Researchers such as Bishop and Glynn (1999) have written about the importance for Maori children of feeling positive about their language and culture. In addition to promoting strong cultural identity, School D also placed a high priority on numeracy and improving achievement levels in mathematics. This could be seen in one of its learning goals for the year, which was "to improve basic facts knowledge and mathematics results." These two factors may have contributed to Maori children at School D having better knowledge of number facts than same-age peers at the other three schools, including the decile 5 school. Superior knowledge of number facts was very evident at School D, where children were consistently better at recalling single-digit doubles (4+4 to 9+9, and 10+10) than children at the other schools. Children at School D also did better than the children in Decile 1 schools on recalling number combinations for ten (e.g., 10 – 1 = ?). (Note: Direct comparison with the decile 5 school was not possible because the tasks were presented as missing addend problems rather than as subtraction problems). Not only did School D put a strong emphasis on learning number facts, but it also provided incentives for children to master these facts by creating a special "club" for children who had gained a perfect score on the test of number facts, or made a substantial improvement in test score from one term to the next.

The disappointing performance of Maori at School A raises some important questions which need addressing. At School A, Maori children were in a minority and many may have been isolated and marginalised. Until quite recently, the option to learn in Te Reo Maori was not available to children at School A. Another possible explanation is that the decile level does not capture the heterogeneity in socio-economic status of the families served by School A. If Maori at School A includes a disproportionately large number of children from disadvantaged families, while constituting only a small proportion of the school roll, then this may no be reflected in the decile level. It is possible that in schools where Maori are in a small minority, their need for additional resources is equal or higher than in schools with a high Maori population where cultural identity is strong. It might be all too easy to overlook those small numbers of Maori children in a large school such as School A with more than 600 pupils.

Children's performance and use of particular strategies was examined according to the level of the framework to which they had been assigned. There was remarkable consistency in the use of strategies within each level. Few, if any, of the children at the Unitary level could use anything other than a counting-by-ones strategy to add or subtract, either mentally or with written problems (presented horizontally), or to construct quantities using grouped materials or different denominations of money. Few, if any, could demonstrate place-value understanding using the boxes task, which required them to demonstrate the link between each digit in a multi-digit numeral and the number of boxes (arranged in rows of ten) to which the digit refers. On the other hand, most children at the Multi-unit level could add or subtract, either mentally or with written problems (presented horizontally), and construct quantities using grouped materials or different denominations of money, using units larger than one. Most were able to solve 2- and 3-digit addition problems with regrouping, recall all single-digit combinations for ten, recall number facts for all doubles with single-digit addends, and count by tens to at least 100. All these children, by definition, had demonstrated place-value understanding with the boxes tasks for all of the problems presented, including the 3-digit number. At the Ten-structured level, many children were able to add or subtract using units larger than one and construct quantities using grouped materials or different denominations of money, recall number facts for doubles with addends up to 6, recall all single-digit combinations for ten except 7 and 3, and count by tens to at least 100. Some of the children at the Ten-structured level were able to demonstrate place-value understanding for some of the multi-digit numbers presented, and a few were able to solve 2- and 3-digit addition problems with regrouping.

The tasks which were used to assign children to one of the three levels of the framework were remarkably effective. The task which involved the addition of imaginary objects, with 20 lollies plus 8 more lollies differentiated the children at the Unitary level from those at higher levels. This task was very simple and quick to give, and the use of a counting-by-ones strategy was easy to identify because of the extra time needed to count 8 or more imaginary lollies. Only one child of the 221 interviewed was able to demonstrate place-value understanding without succeeding on the "20 plus 8" task. That kind of consistency supports the idea that there is a Ten-structured stage that is transitional between Unitary understanding and Multi-unit or Place-value Understanding (Fuson et al, 1997a, 1997b). Helping children to combine and partition quantities without counting by ones, and to appreciate the way that the number system is constructed around groupings of ten may be a vital step towards their eventual understanding of place value. The one child who used counting-by-ones for the "20 plus 8" task, even though she could demonstrate place-value understanding with the all four of the boxes tasks, was able to add and subtract using units larger than one as well as construct quantities using grouped objects and different denominations of money, providing strong support for the idea that she was operating beyond the Unitary level.

It was interesting to note that being able to demonstrate place-value understanding with the boxes task did not necessarily mean that the children were able to use that knowledge to solve problems with regrouping. For example, for the word problems involving multi-digit addition (18 + 15, 29 + 23, and 125 + 85), only about half to two thirds of the children at the Multi-unit level were able to calculate the correct answers. It is possible that some of them decided not to attempt the more difficult "sums" because these were presented at the end of the interview and fatigue had set in. A few of the children at the Ten-structured stage could solve those addition equations with regrouping, but not one of the children at the Unitary level was able to do them.

The tasks which used money of different denominations ($1, $10, $100) suggest a possible direction for teachers who want to use real-life contexts to aid place-value understanding and other related concepts. Most children at the Ten-structured stage were able to assign different values to different money items as part of determining how many dollars there were in a mixed collection of $10 notes and $1 coins. This fits with Baroody's (1990) idea that a different-looking ten like a $10 note, which has no physical relationship to 10 $1 coins, may have advantages over the use of a pre-structured ten like the place-value "long," which is virtually identical to 10 small cubes interlocked. The different-looking appearance of the $10 note compared to the $1 coin means that it is impossible to "see" the 10 ones which are equivalent in value to the $10 note. However, this did not stop one seven-year-old from tapping the $10 note ten times with her finger as she counted from one to ten, before counting on the additional four $1 coins. A substantial number of children at the Unitary level simply added up the number of money items present without regard to the differential value of the notes and coins. It would be interesting to explore the usefulness of money as an aid to solving multi-digit addition and subtraction with regrouping.

There are a number of important implications of these findings for teachers who are trying to help their students understand the number system. This includes the importance of building a strong foundation in Ten-structured understanding prior to expecting children to learn about the significance of position for the value of a particular digit within a multi-digit numeral. Using a range of materials with groupings by ten invites the child to move away from a "by ones" (Unitary) strategy towards combining and partitioning larger units. There are many different resources which can be used to strengthen Ten-structured understanding (see Young-Loveridge, 1999a). For example, embodiments of the empty number line in the form of 100-bead strings composed of groups of ten beads in alternating colours provide a linear representation of numbers. Grouped objects such as plastic beans stored in tens in clear plastic bags can provide a different model for numbers. Money has the added advantage of being highly motivating for children, many of whom have played board games with money. Through activities such as these, children may come to understand place value, not as an isolated piece of knowledge, but as a particular instance of part-whole relationships involving groupings of tens and ones (Fuson, 1992).

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